

HARMONICS PROBLEM

The tallest load-bearing columns are part of the Temple of Amun in Egypt, built in 1270 B.C. Find the height of these columns if a standing wave with a frequency of 47.8 Hz is generated in an open pipe that is as tall as the columns. The sixth harmonic is generated. The speed of sound in air is 334 m/s.

SOLUTION

Given: $f_6 = 47.8 \text{ Hz}$ $v = 334 \text{ m/s}$
 $n = \text{number of the harmonic} = 6$

Unknown: $L = ?$

When the pipe is open, the wavelength associated with the first harmonic (fundamental frequency) is twice the length of the pipe.

$$f_n = n \frac{v}{2L} \quad n = 1, 2, 3, \dots$$

$$L = n \frac{v}{2f_n} = (6) \frac{(334 \text{ m/s})}{2(47.8 \text{ Hz})} = 21.0 \text{ m}$$

ADDITIONAL PRACTICE

1. A 47.0 m alphorn was made in Idaho in 1989. An alphorn behaves like a pipe with one end closed. If the frequency of the fifteenth harmonic is 26.7 Hz, how long is the alphorn? The speed of sound in air is 334 m/s.
2. A fully functional acoustic guitar over 8.0 m in length is on display in Bristol, England. Suppose the speed of waves on the guitar's strings is $5.00 \times 10^2 \text{ m/s}$. If a third harmonic is generated on a string, so that the sound produced in air has a wavelength of 3.47 m, what is the length of the string? The speed of sound in air is 334 m/s.
3. The unsupported flagpole built for Canada's Expo 86 has a height of 86 m. If a standing wave with a 19th harmonic is produced in an 86 m open pipe, what is its frequency? The speed of sound in air is 334 m/s.
4. A power-plant chimney in Spain is $3.50 \times 10^2 \text{ m}$ high. If a standing wave with a frequency of 35.5 Hz is generated in an open pipe with a length equal to the chimney's height and the 75th harmonic is present, what is the speed of sound?
5. The world's largest organ was completed in 1930 in Atlantic City, New Jersey. Its shortest pipe is 4.7 mm long. If one end of this pipe is closed, what is the number of harmonics created by an ultrasound with a wavelength of 3.76 mm?

HARMONICS PROBLEM

A piano wire vibrates with a fundamental frequency of 440 Hz when the speed of sound on the wire is 550 m/s. What is the length of this wire?

SOLUTION

Given: $v = 550 \text{ m/s}$ $n = 1$ $f_1 = 440 \text{ Hz}$

Unknown: $L = ?$

Choose the equation(s) or situation: The fundamental frequency can be found by using the equation for standing waves on a vibrating string:

$$f_n = \frac{nv}{2L}, n = 1, 2, 3, \dots$$

Rearrange the equation(s) to isolate the unknown(s): Rearrange the equation above to solve for the length of the wire.

$$L = \frac{nv}{2f_n} = \frac{(1)(550 \text{ m/s})}{(2)(440 \text{ Hz})} = \boxed{0.625 \text{ m}}$$

ADDITIONAL PRACTICE

- What is the fundamental frequency of a guitar string when the speed of waves on the string is 115 m/s and the effective string lengths are as follows?
 - 70.0 cm
 - 50.0 cm
 - 40.0 cm
- A clarinetist plays a clarinet on a cold day. At one point she produces the sound of middle F sharp, which has a frequency of 370 Hz, by playing the third harmonic of low B. If the speed of sound in the air is 331 m/s, what is the length of the clarinet? Recall that a clarinet resembles a pipe closed at one end.
- A penny whistle plays a tune in the key of G with a fundamental frequency of 392.0 Hz. The speed of sound in air is 331 m/s. What is the length of the penny whistle? Treat the penny whistle as a pipe closed at one end.
- An organ pipe that is open at both ends has a fundamental frequency of 370.0 Hz when the speed of sound in air is 331 m/s. What is the length of this pipe?
- What is the fundamental frequency of a viola string that is 35.0 cm long when the speed of waves on this string is 346 m/s?
- A violin string that is 50.0 cm long has a fundamental frequency of 440 Hz. What is the speed of the waves on this string?
- What is the fundamental frequency of a cello string that is 0.85 m long when the speed of waves on this string is 499 m/s?
- A pipe that is open at both ends has a fundamental frequency of 277.2 Hz. If the pipe is 0.75 m long, what is the speed of the waves in the pipe?
- A pipe that is closed on one end has a seventh harmonic frequency of 466.2 Hz. If the pipe is 1.53 m long, what is the speed of the waves in the pipe?
- A pipe that is open at both ends has a fundamental frequency of 125 Hz. If the pipe is 1.32 m long, what is the speed of the waves in the pipe?